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Solving Linear Programming Problems and Transportation Problems using Excel Solver

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Abstract- This paper outlines the steps required for installing Excel Solver in Microsoft Word 2010 for use in solving linear programming problems it provides a step-by-step procedure with snapshots for improved performance. Several questions are solved including transportation problems using Excel Solver.

Index Terms- Excel Solver, linear programming, maximization, minimization, optimization, profit, transportation problem.

1

Introduction

THE use of Excel Solver for analysis of operations research problems is important and useful in present day technological world. It is difficult to solve linear programming problems using the manual method in organizations that solve problems with over fifty variables. A work that can take days or weeks to solve could be done in a matter of seconds using Excel Solver. Excel Solver has proven to be relevant in other disciplines such as finance, production management, etc. in this paper, I shall present a step-by-step procedure to follow in the installation and use of Excel Solver for solving linear programming problems and transportation problems.

2. Literature Review

Linear Programming

I will skip the definition of terms in linear programming and the assumptions and go straight to problem solving with Excel Solver. It is believed that the reader has prior knowledge of the subject matter. If you haven't installed Excel Solver in your Microsoft Excel, then follow the steps below:

a. Launch Microsoft Excel.

b. Go to "File" click on it and select "Options" (figure 1).

c. A dialog box will be displayed. Select "Add-Ins" (figure 2).

d. Choose "excel solver" and click "Go" and "OK" (figure 3).

e. Close and re-launch Microsoft Excel. Select the "Data" column. You can see "Solver" being displayed (figure 4).

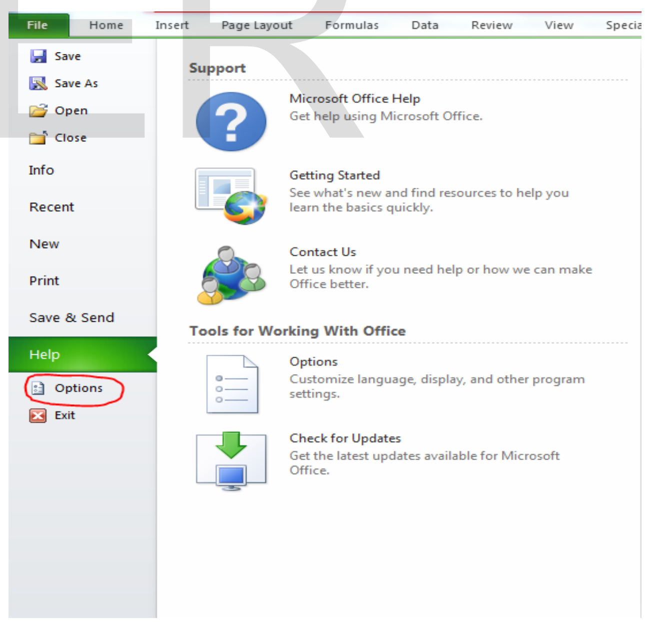

Figure 1

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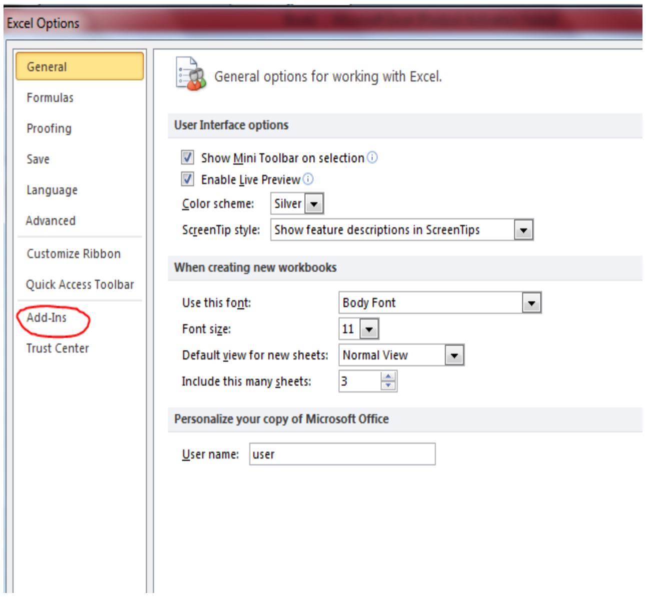


Figure 2

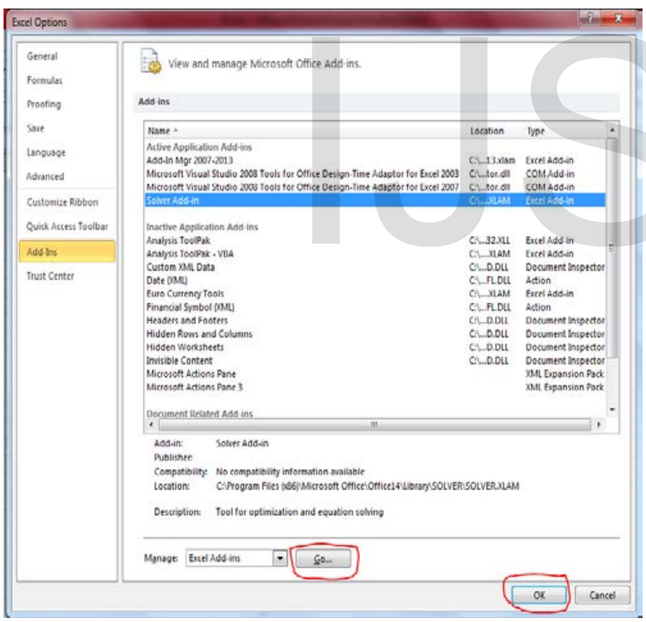


Figure 3

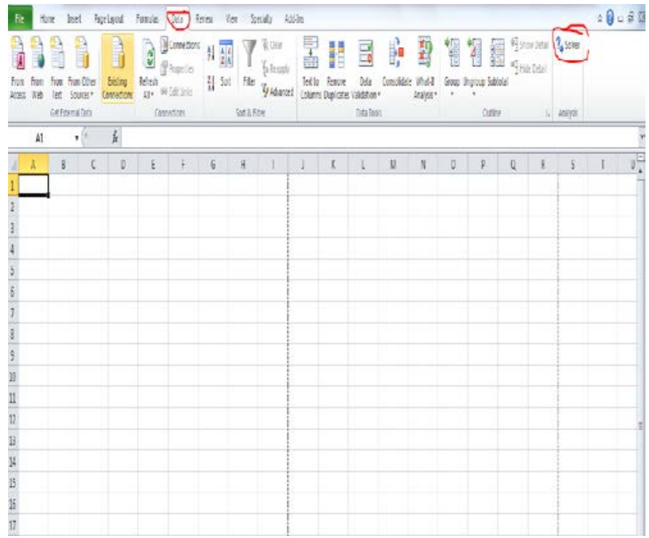


Figure 4

Let's begin with a simple illustration:

Example 1: Max. $z = 20x_1 + 15\,x_2$

s.t.

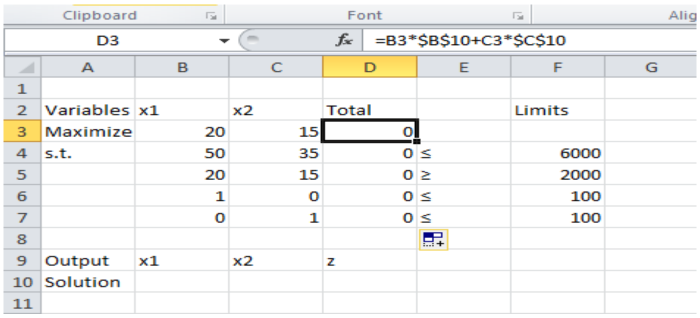
$$50x_1 + 35x_2 \le 6000$$
$$20x_1 + 15x_2 \ge 2000$$
$$x_1 \quad \le 100$$
$$x_2 \le 100$$
$$x_1, x_2 \le 0$$

Input your data into Microsoft excel worksheet as you can see in the figure 5 below. Then add the other items as displayed.

	A	B	C	D	E	F
1						
2	Variables	x1	x2	Total		Limits
3	Maximize	20	15			
4	s.t.	50	35		≤	6000
5		20	15		≥	2000
6		1	0		≤	100
7		0	1		≤	100
8						
9	Output	x1	x2	z		
10	Solution					

Figure 5

In the total column for maximization (i.e. in D3) input the following command: B3*$B$10+C3*$C$10. You can either use upper case or lower case to insert the command. When you are done, click on D3, place the pointer at the lower right hand tip of the cell and drag it down to D7. The formulae for the constraints will be automatically produced. By now, your excel page should look like this:



	A	B	C	D	E	F	G
1							
2	Variables	x1	x2	Total		Limits	
3	Maximize	20	15	0			
4	s.t.	50	35	0	≤	6000	
5		20	15	0	≥	2000	
6		1	0	0	≤	100	
7		0	1	0	≤	100	
8							
9	Output	x1	x2	z			
10	Solution						
11							

Figure 6

You can see the formula on D3 cell being displayed in the formula bar. The formulae for D4 to D7 are:

D4 =b4*b10+c4*c10
D5 =b5*b10+c5*c10

D6 =b6*b10+c6*c10
D7 =b7*b10+c7*c10

You can as well insert them one after the other if it's more convenient.

In cell D10, type "=D3". Now that your data is ready, you solve the linear programming problem using Excel Solver. Click on Data on the menu bar and select Solver.

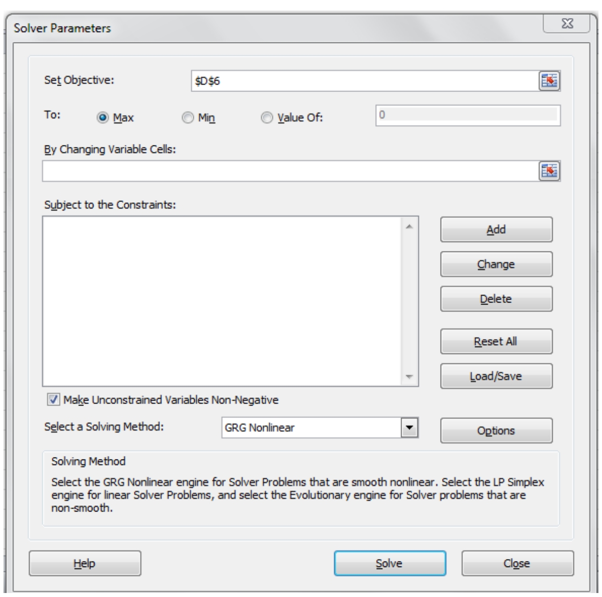


Figure 7

In the objective column, type $D$3. By default, max is selected. In minimization problems, you change to min. In the next column, (i.e. "by changing variable cells") type b10:c10. To insert the constraints, select "Add" (figure 8) and input the following command, the right hand side command on the "Cell reference" box and the lefthand side command on the "Constraint" box. Then select "OK."

$B$10:$C$10≥0
$D$4:$D$7≤$F$4:$F$7

Figure 8

This is how the Solver Parameter should look like after inputing the instructions above:

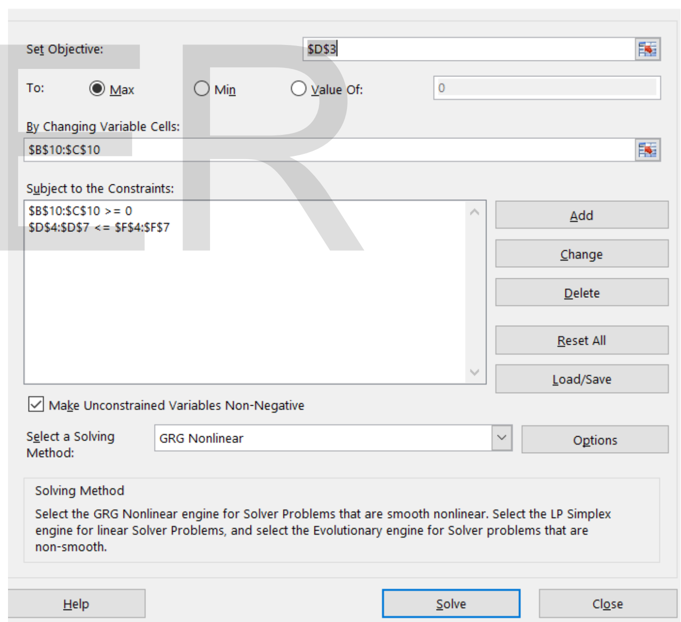


Figure 9

Then click on "Solve". The values of x_1, x_2 and the objective function are: 64, 48 and 2000 respectively. The model and the solution are shown below:

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D3 | f_x =B3*$B$10+C3*$C$10

	A	B	C	D	E	F	G
1							
2	Variables	x1	x2	Total		Limits	
3	Maximize	20	15	2000			
4	s.t.	50	35	4880	≤	6000	
5		20	15	2000	≥	2000	
6		1	0	64	≤	100	
7		0	1	48	≤	100	
8							
9	Output	x1	x2	z			
10	Solution	64	48	2000			

Figure 10

Here is a question for you to practice. Remember to follow the step by step procedure I laid out for you above.

Exercise 1: Min $z= 0.3x_1 + 0.9x_2$

s.t.

$x_1 + x_2 \geq 800$
$0.21x_1 - 0.3x_2 \geq 0$
$0.03x_1 - 0.1x_2 \geq 0$
$x_1, x_2 \geq 0$

The question and solution should look like this:

D4 | f_x =B4*$B$11+C4*$C$11

	A	B	C	D	E	F	G
1	Minimization Problem						
2	Input data:						
3		x1	x2	Totals		Limits	
4	Objective	0.3	0.9	240			
5	s.t.	1	1	800	>=	800	
6		0.21	-0.3	168	>=	0	
7		0.03	-0.1	24	>=	0	
8	Non-neg	>=0	>=0				
9	Output results:						
10		x1	x2	z			
11	Solution	800	0	240			
12							

Figure 11

The non-negativity added is insignificant since it is already included as one of the variables. Did you get the result right? It is very interesting. More exercises will help you master how to solve linear programming problems using Excel Solver with ease.

Now, try this question:

Exercise 2: Max $z= 5x_1 + 4x_2$

s.t.

$6x_1 + 4x_2 \leq 24$
$x_1 + 2x_2 \leq 6$
$-x_1 + x_2 \leq 1$
$x_2 \leq 2$
$x_1, x_2 \leq 0$

The question and solution to the problem in the excel worksheet is given below:

D5 | f_x =B5*$B$13+C5*$C$13

	A	B	C	D	E	F	G
1			Reddy Mikks Model				
2	Input data:						
3		x1	x2				
4		Exterior	Interior	Totals		Limits	
5	Objective	5	4	21	<=		
6	s.t.	6	4	24	<=	24	
7		1	2	6	<=	6	
8		-1	1	-1.5	<=	1	
9		0	1	1.5	<=	2	
10							
11	Output results:						
12		x1	x2	z			
13	Solution	3	1.5	21			

Figure 12

Now, let's solve a real life problem by first formulating the model.

Example 2: Reddy Mikks produces both interior and exterior painnts from two raw materials, M1 and M2. The following table provides the basic data of the problem:

The Reddy Mikks Company

Tons of raw material per tons of			Maximum daily available (tons)
	Exterior paint	Interior paint	
Raw material M1	6	4	24
Raw material M2	1	2	6
Profit per ton ($1000)	5	4	

Table 1

A market survey indicates that the daily demand for interior paint cannot exceed that for exterior piant by more than 1 ton.

Also, the maximum daily demand for interior paint is 2 tons. Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit [Taha (2011), p.47].

Let x_1 represent the number of tons of exterior paints produced and x_2 the number of interior paints produced.

$$\text{Maximize } z = 5x_1 + 4x_2 \text{ (in \$1000)}$$

s.t.

$$6x_1 + 4x_2 \le 24 \text{ (M1)}$$
$$x_1 + 2x_2 \le 6 \text{ (M2)}$$
$$-x_1 + x_2 \le 1 \text{ (Market limit)}$$
$$x_2 \le 2 \text{ (Maximum daily demand)}$$
$$x_1, x_2 \ge 0$$

The solution to the problem is given in figure 13 below.

D5 =B5*$B$13+C5*$C$13

	A	B	C	D	E	F	G
1			Reddy Mikks Model				
2	Input data:						
3		x1	x2				
4		Exterior	Interior	Totals		Limits	
5	Objective	5	4	21	<=		
6	Raw material 1	6	4	24	<=	24	
7	Raw material 2	1	2	6	<=	6	
8	Market limit	-1	1	-1.5	<=	1	
9	Demand limit	0	1	1.5	<=	2	
10		>=0	>=0				
11	Output results:						
12		x1	x2	z			
13	Solution	3	1.5	21			
14							

Figure 13

Now, try this exercise.

Exercise 3: An auto company manufactures cars and trucks. Each vehicle must be processed in the paint shop and body assembly shop. If the paint shop were only painting trucks, then 40 per day could be painted. If the body shop were only producing cars, then it could process 50 per day. Each truck contributes \$300 to profit, and each car contributes \$200 to profit. Use linear programming to determine a daily production schedule that will maximize the company's profit (Winston, 2004).

Solution: Let x_1 and x_2 represent the number of trucks and cars produced respectively.

Fraction of day paint shop works on trucks = (fraction of day/truck) * (trucks/day)

$$= {}^{1}/_{40}x_1$$

Fraction of day body shop works on trucks = ${}^{1}/_{60}x_2$

Fraction of day body shop works on trucks = ${}^{1}/_{50}x_1$

Fraction of day body shop works on cars = ${}^{1}/_{50}x_2$

Hence, the constraints are:

${}^{1}/_{40}x_1 + {}^{1}/_{60}x_2 \le 1$ (Paint shop constraint)
${}^{1}/_{50}x_1 + {}^{1}/_{50}x_2 \le 1$ (Body shop constraint)

The model for the problem is:

$$\text{Max } z = 3x_1 + 2x_2$$

s.t.

$${}^{1}/_{40}x_1 + {}^{1}/_{60}x_2 \le 1$$
$${}^{1}/_{50}x_1 + {}^{1}/_{50}x_2 \le 1$$
$$x_1, x_2 \ge 0$$

Insert the model into a Microsoft Excel worksheet and solve.

TRANSPORTATION PROBLEM

Transportation problems can be solved using Excel Solver. What is required is to change the problem into a linear programming problem and solve it as a minimization problem following the same procedure as explained above. Before you proceed, you may need to study transportation problem first for better understanding.

Excel Solver and TORA can be used for solving different forms of transportation problem. Excel Solver can only compute the least transportation cost without giving credence to its computation using three methods: Least Cost Method, North West Corner Method and Vogel Approximation; which are exemplified by TORA. This is made possible because the problem is first changed to a LP problem and solved using the simplex method. According to Taha (2011)," TORA handles all necessary computations in the background using the simplex method and uses the transportation model format only as a screen 'veneer'". The two methods, however, do not solve transportation problems using the MODI method.

Example 1: MG Auto has three plants in Los Angeles, Detroit, and New Orleans, and two major distribution centers in Denver and Miami. The quarterly capacities of the three plants are 1000, 1500, and 1200 cars, and the demands at the two distribution centers for the same period for the same period are 2300 and 1400 cars [Taha (2011), p.209].

	Denver	Miami	Supply
Los Angeles	80	215	1000
Detroit	100	108	1500
New Orleans	102	68	1200
Demand	2300	1400	

This problem can be changed to a linear programming problem as follows:

Minimize $Z = 80x_{11} + 215x_{12} + 100x_{21} + 108x_{22} + 102x_{31} + 68x_{32}$

Subject to:

$x_{11} + x_{12} \geq 1000$ (Los Angeles)

$x_{21} + x_{22} \geq 1500$ (Detroit)

$x_{31} + x_{32} \geq 1200$ (New Orleans)

$x_{11} + x_{21} + x_{31} \geq 2300$ (Denver)

$x_{12} + x_{22} + x_{32} \geq 1400$ (Miami)

Insert the model into an excel worksheet. This is what you should have:

H4 | fx | =B4*$B$12+C4*$C$12+D4*$D$12+E4*$E$12+F4*$F$12+G4*$G$12

	A	B	C	D	E	F	G	H	I	J	K
1	MG Transportation Problem (Converted to a LP problem)										
2	Input										
3		x11	x12	x21	x22	x31	x32	Total		Limits	
4	Objective	80	215	100	108	102	68	0			
5	Constraint	1	1	0	0	0	0	0	>=	1000	
6	Detriot	0	0	1	1	0	0	0	>=	1500	
7	New Orle	0	0	0	0	1	1	0	>=	1200	
8	Denver	1	0	1	0	1	0	0	>=	2300	
9	Miami	0	1	0	1	0	1	0	>=	1400	
10											
11	Ouput	x11	x12	x21	x22	x31	x32	Z			
12	Result							0			
13											

Figure 14

H4 was highlighted. Insert the formula for H4 and drag it down to H9. Then in cell H12 type " =H4". Then go to Solver Parameter to solve the problem. With the help of the examples above, the Solver Parameter for this question should look like this:

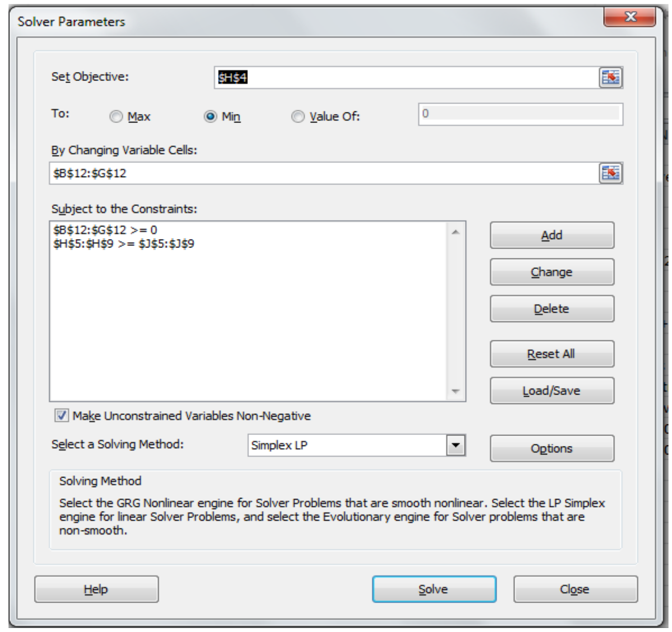

Figure 15

Click on "Solve" when you have supplied the values. Your result will show in the worksheet as you can see in figure 16 below.

H4 | fx | =B4*$B$12+C4*$C$12+D4*$D$12+E4*$E$12+F4*$F$12+G4*$G$12

	A	B	C	D	E	F	G	H	I	J	K
1	MG Transportation Problem (Converted to a LP problem)										
2	Input										
3		x11	x12	x21	x22	x31	x32	Total		Limits	
4	Objective	80	215	100	108	102	68	313200	>=		
5	Constraints: LA	1	1	0	0	0	0	1000	>=	1000	
6	Detriot	0	0	1	1	0	0	1500	>=	1500	
7	New Orleans	0	0	0	0	1	1	1200	>=	1200	
8	Denver	1	0	1	0	1	0	2300	>=	2300	
9	Miami	0	1	0	1	0	1	1400	>=	1400	
10											
11	Ouput	x11	x12	x21	x22	x31	x32	Z			
12	Result	1000	0	1300	200	0	1200	313200			
13											

Figure 16

The result shows that 1000 units of the product should be shipped to Denver from Los Angeles, 1300 units from Detriot

to Denver, 200 units from Detroit to Miami and 1200 units from New Orleans to Miami to minimize cost.

Excel Solver only features the least cost method thus it cannot solve the question using North-West Corner Method or Vogel Approximation method. However, TORA software features all three methods.

Below is a transportation question you can solve and the solution there in.

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Exercise 1: Powerco has three electric power plants that supply the needs of four cities. Each power plant can supply the following numbers of kilwatt-hours (kwh) of electricity: plant 1-35million; plant 2-50 million; plant 3-40 million. The peak power demands in these cities, which occur at the same time (2p.m.), are as follows (in kwh): city 1-45 million; city 2-20 million; city 3-30 million; city 4-30 million. The costs of sending 1 million kwh of electricity from plant to city depend on the distance the electricity must travel. The cost of shipping is shown in the table 1 below. Formulate an LP to minimize the cost of meeting each city's peak power demand (Winston, 2004, p.360).

From	To			
	City 1	City 2	City 3	City 4
Plant 1	$8	$6	$10	$9
Plant 2	$9	$12	$13	$7
Plant 3	$14	$9	$16	$5

Table 1

Solution

The shipping cost, supply and demand for power is shown in table 2 below.

Table 2

Xij= number of (million) kwh produced at plant i and sent to city j (where i=1,2,3 and j=1,2,3,4)

Max z= $8x_{11} + 6x_{12} + 10x_{13} + 9x_{14} + 9x_{21} + 12x_{22} + 13x_{23} + 7x_{24} + 14x_{31} + 9x_{32} + 16x_{33} + 5x_{34}$

Subject to:

Supply constraints:

$$x_{11} + x_{12} + x_{13} + x_{14} \le 35$$
$$x_{21} + x_{22} + x_{23} + x_{24} \le 50$$
$$x_{31} + x_{32} + x_{33} + x_{34} \le 40$$

Demand constraints:

$$x_{11} + x_{21} + x_{31} \le 45$$
$$x_{12} + x_{22} + x_{32} \le 20$$
$$x_{13} + x_{23} + x_{33} \le 30$$
$$x_{14} + x_{24} + x_{34} \le 30$$

Insert the model into an excel worksheet. Then go to the "solver parameter" and input the required command as shown in figure 4 below.

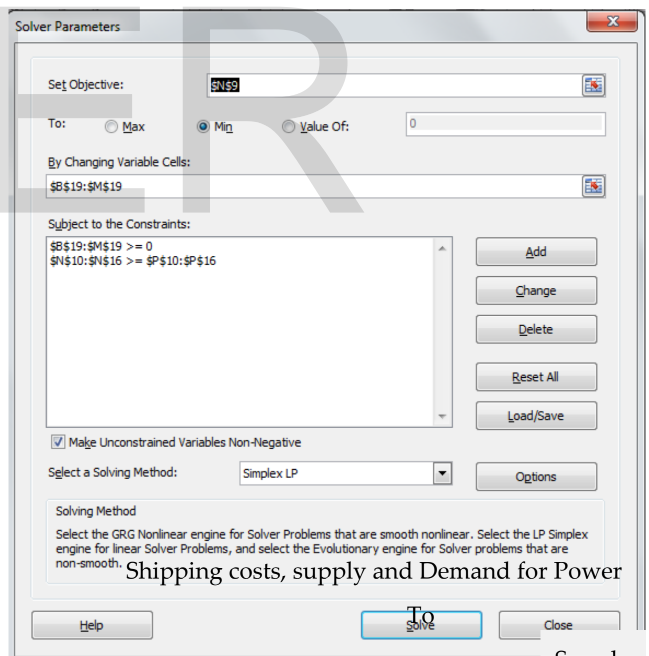


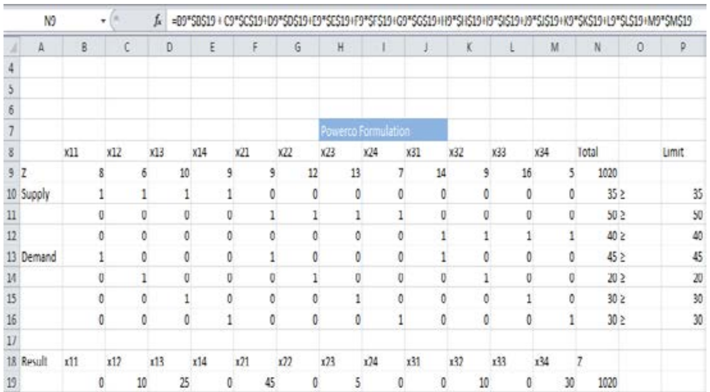
Figure 4

The model and solution to

Shipping costs, supply and Demand for Power

From	To				Supply (million kwh)
	City1	City 2	City 3	City 4	
Plant 1	8	6	10	9	35
Plant 2	9	12	13	7	50
Plant 3	14	9	16	5	40
Demand (million kwh)	45	20	30	30	

the problem are shown in table 5 below.



N9 =B9*$B$19+C9*$C$19+D9*$D$19+E9*$E$19+F9*$F$19+G9*$G$19+H9*$H$19+I9*$I$19+J9*$J$19+K9*$K$19+L9*$L$19+M9*$M$19

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
4																
5																
6																
7								Powerco Formulation								
8		x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	Total		Limit
9	Z	8	6	10	9	9	12	13	7	14	9	16	5	1020		
10	Supply	1	1	1	1	0	0	0	0	0	0	0	0	35	≥	35
11		0	0	0	0	1	1	1	1	0	0	0	0	50	≥	50
12		0	0	0	0	0	0	0	0	1	1	1	1	40	≥	40
13	Demand	1	0	0	0	1	0	0	0	1	0	0	0	45	≥	45
14		0	1	0	0	0	1	0	0	0	1	0	0	20	≥	20
15		0	0	1	0	0	0	1	0	0	0	1	0	30	≥	30
16		0	0	0	1	0	0	0	1	0	0	0	1	30	≥	30
17																
18	Result	x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	7		
19		0	10	25	0	45	0	5	0	0	10	0	30	1020		

Figure 5

The formula for cell N9 is shown on the formula box. Always check your input to confirm it is right before solving the model. Optimal solution to this LP is z= 1020, x_{12}=10, x_{13}=25, x_{21}=45, x_{23}=5, x_{32}=10, x_{34}=30.

This method of computation involves the generation of variables that make the calculations cumbersome as the demand and supply centers increase.

Let's try a different method for solving transportation problem below:

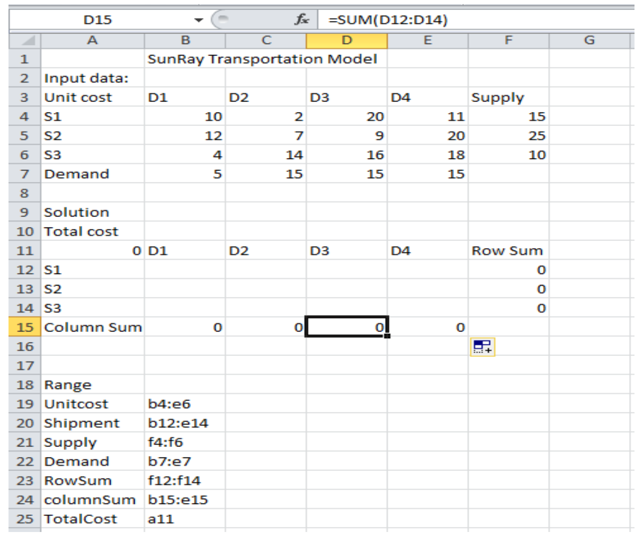
Example 2: SunRay Transport Company ships truckloads of grain from three silos to four mills. The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckload on the different routes are summarized in table . the unit transportation costs (shown in the northeast cortner of each box) are in hundreds of dollars. The model seeks the minimum-cost shipping schedule between the silos and the mills.

SunRay Transportation Model

	Mill				
	1	2	3	4	Supply
1	10	2	20	11	15
Silo 2	12	7	9	20	25
3	4	14	16	18	10
Demand	5	15	15	15	

Table 3

Fill in the information as shown in figure below. To fill in the range names to cells, select the appropriate cell and right click.



D15 =SUM(D12:D14)

	A	B	C	D	E	F	G
1		SunRay Transportation Model					
2	Input data:						
3	Unit cost	D1	D2	D3	D4	Supply	
4	S1	10	2	20	11	15	
5	S2	12	7	9	20	25	
6	S3	4	14	16	18	10	
7	Demand	5	15	15	15		
8							
9	Solution						
10	Total cost						
11	0	D1	D2	D3	D4	Row Sum	
12	S1					0	
13	S2					0	
14	S3					0	
15	Column Sum	0	0	0	0		
16							
17							
18	Range						
19	Unitcost	b4:e6					
20	Shipment	b12:e14					
21	Supply	f4:f6					
22	Demand	b7:e7					
23	RowSum	f12:f14					
24	columnSum	b15:e15					
25	TotalCost	a11					

Figure 18

Double-click on any cell. Select "define name" and a dialog box like the one in figure 19 appears. Fill in the necessary details as shown on the range items in table below.

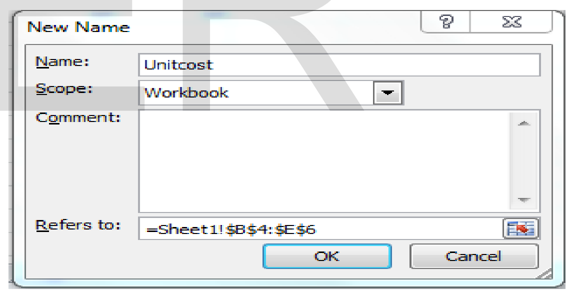


Figure 19

Open the Excel Solver and insert the information in the appropriate order as shown in figure 20 below.

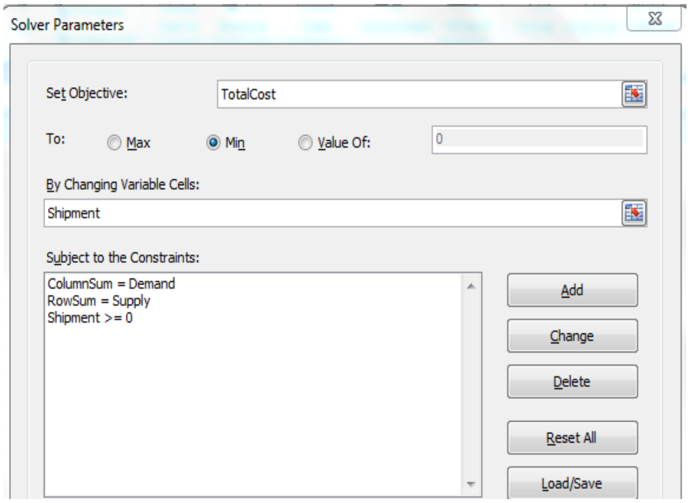


Figure 20

Click on Solve. The solution to the problem is displayed in the worksheet as you can see below in figure 21.

	SunRay Transportation Model				
Input data:					
Unit cost	D1	D2	D3	D4	Supply
S1	10	2	20	11	15
S2	12	7	9	20	25
S3	4	14	16	18	10
Demand	5	15	15	15	
Solution					
Total cost					
435	D1	D2	D3	D4	Row Sum
S1	0	5	0	10	15
S2	0	10	15	0	25
S3	5	0	0	5	10
Column Sum	5	15	15	15	

References

[1] Taha, H. A. (2011). Operations research: An introduction. 9th Ed. Pearson: New Jersey

[2] Winston, W. L. (2004). Operations research: Applications and algorithms (4th ed.) Brooks: Canada